Weinberg 3.1

$$(\Phi_\beta, V\Phi_\alpha) = g\, u_\beta u_\alpha^* \quad , \quad \sum_\alpha |u_\alpha|^2 = 1$$

$\Uparrow$

This implies discrete set of states.

The Lippmann-Schwinger eq. is general:

$$\Psi_\alpha^\pm = \Phi_\alpha + (E_\alpha - H_0 \pm i\epsilon)^{-1} V \Psi_\alpha^\pm \qquad \text{(Lipp-Schwinger)}$$

expanding (projecting onto) to complete set of free states:

$$\Psi_\alpha^\pm = \Phi_\alpha + \sum_\beta \frac{(\Phi_\beta, V\Psi_\alpha^\pm)}{E_\alpha - E_\beta \cancel{\pm i\epsilon}} \Phi_\beta .$$

Expanding perturbatively:

$$\Psi_\alpha^\pm = \Phi_\alpha + \sum_\beta \frac{1}{E_\alpha - E_\beta \cancel{\pm i\epsilon}} \left(\Phi_\beta, V\left(\Phi_\alpha + \sum_\sigma \frac{(\Phi_\sigma, V\Psi_\alpha^\pm)}{E_\alpha - E_\sigma \cancel{\pm i\epsilon}} \Phi_\sigma \right)\right) \Phi_\beta$$

$$= \Phi_\alpha + \sum_\beta \frac{1}{E_\alpha - E_\beta} (\Phi_\beta, V\Phi_\alpha)\Phi_\beta + \frac{1}{E_\alpha - E_\beta} \sum_\sigma \left(\Phi_\beta, V\frac{(\Phi_\sigma, V\Psi_\alpha^\pm)}{E_\alpha - E_\sigma}\right) \Phi_\beta .$$

$$= \underset{\substack{\uparrow \\ \text{0th order}}}{\Phi_\alpha} + \underset{\substack{\uparrow \\ \text{1st order}}}{\sum_\beta \frac{1}{E_\alpha - E_\beta} (\Phi_\beta, V\Phi_\alpha)\Phi_\beta} + \underset{\substack{\uparrow \\ \text{2nd order}}}{\sum_\beta \sum_\sigma \frac{1}{E_\alpha - E_\beta} \frac{1}{E_\alpha - E_\sigma} (\Phi_\beta, V\Phi_\sigma V\Psi_\alpha^\pm)\Phi_\beta}$$

This expansion can be carried out by replacing $\Psi_\alpha^\pm$ with

$$\Psi_\alpha^\pm = \Phi_\alpha + \sum_\beta \frac{1}{E_\alpha - E_\beta} (\Phi_\beta, V \Psi_\alpha^\pm) \Phi_\beta .$$

Up to first order, it is

$$\Psi_\alpha^\pm = \Phi_\alpha + \sum_\beta \frac{1}{E_\alpha - E_\beta} (\Phi_\beta, V \Phi_\alpha) \Phi_\beta$$

$$= \boxed{\Phi_\alpha + g \sum_\beta \frac{1}{E_\alpha - E_\beta} u_\beta u_\alpha^* \Phi_\beta}$$

I believe this is the result found in QM textbooks.

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